ECE466: Mentored Teaching Project

Assessing the effect of teaching the Radix-3 variant of Fast Fourier Transform (FFT) in students' ability to extrapolate to arbitrary length FFTs

Pedro da Cunha Nariyoshi
Faculty Mentor: Dr. Selin Aviyente

1 Teaching and Learning Goal

In engineering education, students are exposed to to a myriad of algorithms. Although learning algorithm implementations is an useful skill, many times the thinking behind different algorithms is often of even higher importance, as engineers must often create, adapt or extend algorithms to the specific problembeing worked on.

This project aims at evaluating how well students are able to extrapolate a particular set of algorithms taught in Digital Signal Processing. Particularly, the goal is to evaluate if spending additional time teaching a second variant of the algorithm yields any measurable improvement in learning.

2 Teaching Question

One of the most fundamental mathematical tools in digital signal processing is the Discrete Fourier Transform (DFT). The popularity of the DFT is in part due to a set of computationally efficient algorithms to evaluate the DFT known as Fast Fourier Transform (FFT). While the FFT can be implemented for arbitrary lengths, it is usually only to teach variants of the algorithm where the length is a powers of 2 (radix-2 FFT). While this is the most common implementation of the algorithm, this can lead to students simply memorizing this variant, instead of fully comprehending the algorithm. The goal of this exercise is to compare student's ability to extrapolate the knowledge into any arbitrary length if only presented with only the radix-2 FFT to being presented both radix-2 and radix-3 variants.

The findings of this project can be used to guide the teaching of the FFT in future semesters. If the improvement of being presented both radix-2 and radix-3 variants is significant, an argument can be made to spend additional time developing both variants. On the other hand, if the improvements are small, the additional time can be better spent on other topics.

3 Assessment Technique

An in-class activity was designed to evaluate whether presenting both radix-2 and radix-3 increases the likelihood of students' ability to extend the algorithm to arbitrary lengths. First, the radix-2 variant of the FFT was derived with students and the butterfly diagram for length 8 was presented. Afterwards, students were asked to derive the weights for a FFT of length 6 and fill a butterfly diagram. Immediately following the exercise, the radix-3 FFT was derived with a butterfly diagram demonstration for a length 9 FFT. Finally, the previous exercise is reapplied.

Although both radix-2 and radix-3 variants will be presented, the assignment requires the use of a mixed-radix variant, which was not directly shown to students. This will challenge students to write down their own expression for a FFT expansion and 7

The assignments were graded and assigned into one of four categories: 1. First attempt was already correct. 2. First attempt incorrect, but second attempt was correct. 3. First attempt incorrect, second attempt showed improvement, but not fully correct. 4. Both attempts incorrect.

The number of students in each category will be analyzed to determine the effectiveness of the intervention. The second attempt allows students to reflect longer about the exercise and potentially figure out the correct answer regardless of the intervention. To mitigate that, the intervention is applied immediately after the first attempt and the second attempt will be given a reduced amount of time.

4 Classroom Practice

Students will receive two copies of the same assignment stapled together. Although the assignment are anonymous, the two copies are joined to evaluate the performance before and after the intervention. Due to the anonymity, all students that participated in the exercise received 10 extra-credit points (approximately 1.5% of their final grade, same weight of the in-class quizzes) which were added to their final grade.

An option not to participate in the exercise, and instead complete a short programming exercise was provided, but no students chose the alternative assignment (with the exception of one student who missed the lecture).

After the derivation of the radix-2 FFT and the butterfly diagram for a FFT of length 8 was presented, students were given 15 minutes to fill the first copy of the assignment. Then, the radix-3 FFT was derived and the butterfly diagram for a FFT of length 9 was presented. Finally, students were given 10 minutes to complete the second copy of the assignment.

5 Summary of Results

Out of the 17 students, 14 students were present in the lecture for the assignment. All students agreed to participate in the study. The aggregate results of the study are found in table 5.

	Both correct	2nd attempt correct	2nd showed improvement	Remained wrong
# of students	2	7	3	2
Percentage	14.3%	50%	21.4%	14.3%

Table 1: Results of the exercise

In the first attempt, only 14.3% of the students were able to complete the exercise correctly. In the second attempt 64.3% of the students completed the exercise correctly, while an additional 21.4% of the students showed some improvement.

The large proportion of students who answered the second attempt correctly compared to the first attempt indicates that the intervention was successful. Additionally, the large proportion of students who were not able to correctly answer the exercise possibly points to an insufficient time for students to solidify the knowledge. If more time was given to students to complete the exercise, it is likely that more would have been able to succeed.

6 Conclusion

The intervention was successful at increasing the ability of the student group to correctly derive the butterfly diagram weights for a FFT with mixed-radix. This indicates that more students were able to extrapolate from the radix-2 and radix-3 variants of the FFT.

Although it is unlikely that many of the students will be tasked with implementing a FFT routine on their own, the ability to extend and adapt algorithms are very valuable skills that are difficult to teach and exercise.

Even though the intervention was simply an exposition on one additional variant of the FFT, surprisingly, the effects were large, with 58.3% of students who answered the exercise incorrectly on the first attempt being able to answer it correctly after the intervention.

As a result, next time I have the opportunity to present this topic, I intend to present both radix-2 and radix-3 variants as well use a similar in-class activity. I believe an even larger number of students would be able to complete the exercise correctly if other techniques such as peer instruction were applied or more time between attempts was allotted.

A Artifacts

The handout used in the exercise is attached in the following pages.

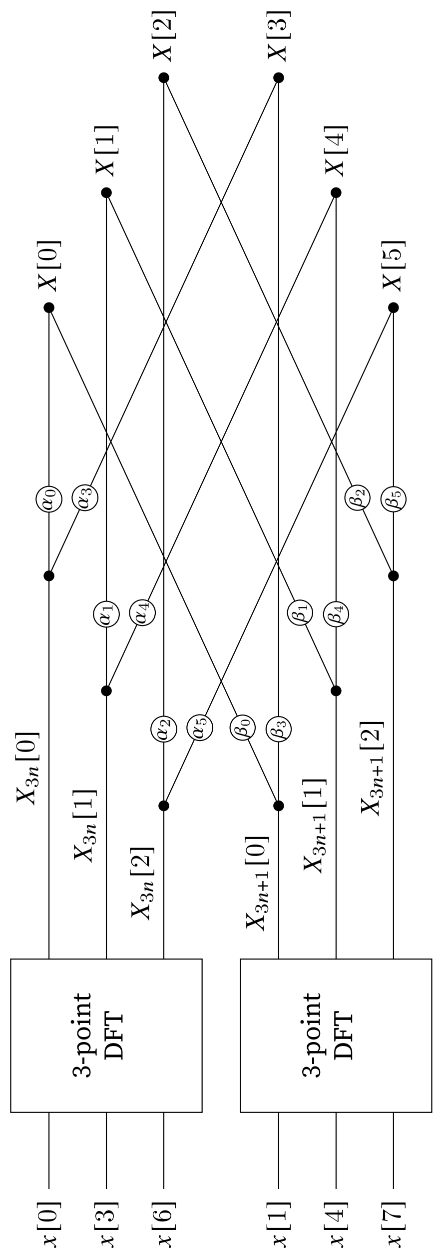
FFT exploration

ECE466: Digital Signal Processing

Task

In this exercise, you will compute a 6-point Fast Fourier Transform (FFT). First, you will watch the first part of the lecture on the FFT. Then, you will be asked to compute weights for the 6-point FFT, the diagram is printed in the next page. You will have 15 minutes to complete the exercise. Afterwards, you will watch the second part of the lecture on the FFT and attempt to compute the 6-point FFT again. Should you choose to participate in this activity, you will be awarded 10 points as a reward, otherwise, you will be assigned an alternative MATLAB-based assignment worth 10 points.

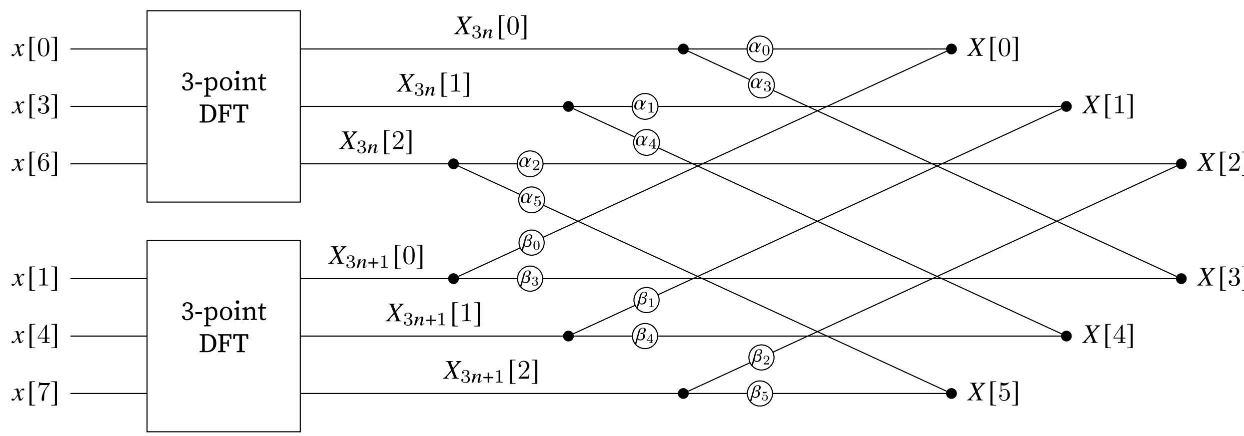
The space below is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.



$\alpha_0 =$ ______ $\alpha_3 =$ ______ $\beta_0 =$ ______ $\beta_3 =$ ______

$\alpha_1 =$ ______ $\alpha_4 =$ ______ $\beta_1 =$ ______ $\beta_4 =$ ______

$\alpha_2 =$ ______ $\alpha_5 =$ ______ $\beta_2 =$ ______ $\beta_5 =$ ______



$\alpha'_0 =$ ______ $\alpha'_3 =$ ______ $\beta'_0 =$ ______ $\beta'_3 =$ ______

$\alpha'_1 =$ ______ $\alpha'_4 =$ ______ $\beta'_1 =$ ______ $\beta'_4 =$ ______

$\alpha'_2 =$ ______ $\alpha'_5 =$ ______ $\beta'_2 =$ ______ $\beta'_5 =$ ______